Appliance of Pumping Data of Wells for Obtaining Transmissivity Distributions of Aquifers for Hydrogeological Model of Latvia

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Abstract – In 2010 – 2012 the hydrogeological model (HM) of Latvia called LAMO was developed by the scientists of Riga Technical University (RTU). LAMO generalizes geological and hydrogeological information accumulated by the Latvian Environment, Geology and Meteorology Centre (LVGMC). The commercial program Groundwater Vistas (GV) was used for running LAMO. In 2013 – 2014 LAMO was considerably upgraded. Density of the hydrogeological network (rivers and lakes) was increased, cuttings of river valleys into primary geological strata were accounted for, transmissivity distributions for aquifers were refined. To improve transmissivity data of HM aquifers, information provided by pumping tests for wells was used. The refined transmissivity data were applied to create the permeability maps of aquifers as the variable initial data for the GV system. To accomplish these task methods of numerical interpolation and digital image processing were used.

Keywords – Hydrogeological model, numerical interpolation, pumping tests for wells, transmissivity of aquifers.

I. INTRODUCTION

In 2010 – 2012 the HM LAMO was developed and in 2013 – 2014 it was upgraded [1] by the scientists of RTU. LAMO comprises the active groundwater zone that provides drinking water. The location of LAMO is shown in Fig. 1. As it follows from the vertical schematization of HM (Fig. 2), the current version of LAMO simulates 27 geological layers, 12 of which are aquifers. As see in Fig. 3 and Fig. 4, most of the layers are outcropping.

They are not continuous and, for this reason, they are not present everywhere in the HM area. After emerging at the surface such layers have zero thickness \( m = 0 \). To avoid in GV calculations “the division by zero”, \( m = 0 \) must be replaced by small \( \varepsilon > 0 \) (for LAMO, \( \varepsilon = 0.02 \) meters). It is explained later that the presence of the \( m = 0 \) areas causes problems when the permeability maps for aquifers are obtained.

To understand the aquifer transmissivity role for HM, basic mathematical expressions of numerical hydrogeological modelling [2] must be considered.

Vector \( \varphi \) of the piezometric head is the numerical solution of the boundary field problem which is approximated in nodes of the HM \( xyz \)-grid by the following algebraic expression:

\[
A \varphi = \beta - G \psi,
\]

\[
A = A_{xy} + A_{z},
\]

(1)

where \( A \) is the symmetric sparse matrix of the geological environment which is presented by the \( xy \)-layer system containing horizontal \((A_{xy} - \text{transmissivity} T)\) and vertical \((A_{z} - \text{vertical hydraulic conductivity})\) elements of the HM grid and \( \psi \) and \( \beta \) are the boundary head and flow vectors, respectively; \( G \) is the diagonal matrix (part of \( A \)) assembled by elements linking the nodes, where \( \varphi \) must be found with the points where \( \psi \) is given.

The transmissivity elements \( a_{xy} \), of \( A_{xy} \), of the HM \( xy \)-planes are computed, as follows:

\[
a_{xy} = k_{i} m_{i} = T_{i}, \quad m_{i} = z_{i-1} - z_{i}, \quad m_{i} > 0, \quad i = 1, 2, \ldots, p
\]

(2)

where

\( z_{i-1} \) and \( z_{i} \) elevations, accordingly, of the top and bottom surfaces of the \( i \)-th geological layer;

\( p \) number of surfaces (for LAMO, \( p = 28 \));

\( z_{0} \) ground surface elevation \( y_{surf} \) map;

\( k_{i}, m_{i} \) elements of the digital \( km \)-maps of the \( i \)-th layer permeability and computed thickness.

The \( m \)-maps are obtained from the \( z \)-maps and the elements \( T_{i} \) are also computed within the GV system [3]. For this reason, it is difficult to change the \( m \)-maps and one has to apply the variable permeability \( k \)-maps to control elements \( T_{i} = a_{xy} \).

The matrices \( A_{xy} \) (transmissivity \( T \)) for aquifers are very important, because they control the horizontal groundwater flow regime.

For aquitards \( a_{xy} \sim 0 \), because their permeability \( k \) is very small and the effect of \( A_{xy} \) is insignificant.
In the Appendix, as an example, obtaining of the \( k \)-maps for the D3pl aquifer is explained.

<table>
<thead>
<tr>
<th>No of HM plane</th>
<th>Name of layer</th>
<th>Geological code</th>
<th>HM plane code</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Relief</td>
<td>relh</td>
<td>relh</td>
</tr>
<tr>
<td>2.</td>
<td>Aeration zone</td>
<td>aer</td>
<td>aer</td>
</tr>
<tr>
<td>3.</td>
<td>Unconfounded Quaternary</td>
<td>Q4-3</td>
<td>Q2</td>
</tr>
<tr>
<td>4.</td>
<td>Upper moraine</td>
<td>gQ3</td>
<td>gQ2z</td>
</tr>
<tr>
<td>5.</td>
<td>Confined Quaternary or Jura</td>
<td>Q1-3</td>
<td>Q1#</td>
</tr>
<tr>
<td>6.</td>
<td>Lower moraine or Triass</td>
<td>gQ1-3</td>
<td>gQ1#z</td>
</tr>
<tr>
<td>7.</td>
<td>Perma Karbons Skerves Keteru</td>
<td>P2</td>
<td>D3ktl#</td>
</tr>
<tr>
<td>8.</td>
<td>Keteru</td>
<td>D3ktl</td>
<td>D3ktlz</td>
</tr>
<tr>
<td>9.</td>
<td>Zgasces Svetes Tervetes Muru</td>
<td>D3g2</td>
<td>D3g2#</td>
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<tr>
<td>10.</td>
<td>Akmenes</td>
<td>D3ak</td>
<td>D3akz</td>
</tr>
<tr>
<td>11.</td>
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<td>D3ak</td>
<td>D3krs#</td>
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<tr>
<td>12.</td>
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<td>D3el</td>
<td>D3elz</td>
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<td>13.</td>
<td>Stipinu Katesu Ogres Daugavas</td>
<td>D3stp</td>
<td>D3dg#</td>
</tr>
<tr>
<td>14.</td>
<td>Daugavas Salspils</td>
<td>D3cg</td>
<td>D3cgp#</td>
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<td>D3plz</td>
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<td>D3am</td>
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</tr>
<tr>
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<td>D3gj2</td>
<td>D3gj2z</td>
</tr>
<tr>
<td>19.</td>
<td>Upper Gauja</td>
<td>D3gj2</td>
<td>D3gj2</td>
</tr>
<tr>
<td>20.</td>
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<td>D3gj1</td>
<td>D3gj1z</td>
</tr>
<tr>
<td>21.</td>
<td>Lower Gauja</td>
<td>D3gj1</td>
<td>D3gj1</td>
</tr>
<tr>
<td>22.</td>
<td>Burtniek Burtnieku</td>
<td>D2brt</td>
<td>D2brtz</td>
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<td>23.</td>
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<td>D2brt</td>
<td>D2brtz</td>
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<tr>
<td>24.</td>
<td>Arikula</td>
<td>D2ar</td>
<td>D2arz</td>
</tr>
<tr>
<td>25.</td>
<td>Arikula</td>
<td>D2ar</td>
<td>D2ar</td>
</tr>
<tr>
<td>26.</td>
<td>Narvas Narvas</td>
<td>D2mn2</td>
<td>D2mnz</td>
</tr>
<tr>
<td>27.</td>
<td>Pernavas</td>
<td>D2pr</td>
<td>D2prz</td>
</tr>
</tbody>
</table>

\* aquitard
\*# = united aquifer; \*#z = united aquitard

Fig. 2. Vertical schematization of LAMO.

Fig. 3. Boundaries of primary geological strata.

Fig. 4. Geological cross section.

II. APPLICATION OF PUMPING TEST RESULTS FOR REFINING TRANSMISSIVITY DATA

For the well pumping test in the confined aquifer the discharge rate \( Q \) was applied and the drawdown \( S \) of the groundwater head was observed. Mathematically the test is presented by the expression \([4]\):

\[
S = \frac{Q}{2\pi R}(\ln(R/r) + \xi), \quad T = \text{km}
\]

where \( R \) are \( r \)-radiuses, accordingly, of the well depression cone and screen, \( \xi \) is additional hydraulic resistance that accounts for the partial penetrating factor of a well. From (3) one can obtain:

\[
T = \frac{q}{2\pi}(\ln(R/r) + \xi), \quad q = \frac{Q}{S}
\]

where \( q \) is the well specific capacity of the well.

If \( q \) and \( T \) have the dimensions, \([\text{litre/sec.meter}]\), and \([\text{meter}^2/\text{day}]\), respectively, then

\[
T = 13.75q(\ln(R/r) + \xi).
\]
All aquifers of LAMO for the primary strata are leaky and confined. Then \( R = 1.12B \) [5]:

\[
B = \sqrt{\frac{km}{k_1/m_1 + k_2/m_2}} \tag{6}
\]

where \( B \) is the leakage factor, \( T = km \) – transmissivity of the aquifer, \( k_1, m_1, \) and \( k_2, m_2 \) are permeability and thickness of the leaky confining aquitards, accordingly, located above and below the aquifer. In Table I values of \( B, R \) and \( \ln(R/r) \) are given for a typical leaky confined aquifer of LAMO when \( r = (0.05 – 0.1) \) meter.

<table>
<thead>
<tr>
<th>( km )</th>
<th>( k_1,k_2 )</th>
<th>( m_1,m_2 )</th>
<th>( B )</th>
<th>( R )</th>
<th>( \ln(R/r) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>10 (^{-4})</td>
<td>10</td>
<td>1 581</td>
<td>1 771</td>
<td>1.12</td>
</tr>
<tr>
<td>100</td>
<td>10 (^{-4})</td>
<td>5</td>
<td>2 236</td>
<td>2 504</td>
<td>0.10</td>
</tr>
<tr>
<td>100</td>
<td>10 (^{-4})</td>
<td>10</td>
<td>1 581</td>
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<td>100</td>
<td>10 (^{-4})</td>
<td>5</td>
<td>2 236</td>
<td>2 504</td>
<td>0.05</td>
</tr>
</tbody>
</table>

By exploiting the fact that in Table I \( \ln(R/r) \sim 10, \) one can approximate formula (5). If \( \xi = 0 \) then the following formula roughly provides the minimal value \( T_{\text{min}} \) of transmissivity for the confined aquifer:

\[
T_{\text{min}} = 137.5q \tag{7}
\]

In [4] the following formula is presented for computing of the resistance \( \xi \):

\[
\xi = (1/a - 1)(\ln 1.47ab – 2.65a), \quad a = l/m, \quad b = m/r \tag{8}
\]

where \( m \) is the thickness of aquifer and \( l \) and \( r \) are, accordingly, the length and radius of the well screen. The formula can be used if \( m/r > 100, \) \( l/m \geq 0.1 \). In Table II the results given by (8) are presented.

<table>
<thead>
<tr>
<th>( l/m )</th>
<th>( m/r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>21.80</td>
</tr>
<tr>
<td>0.2</td>
<td>11.40</td>
</tr>
<tr>
<td>0.3</td>
<td>6.98</td>
</tr>
<tr>
<td>0.4</td>
<td>4.52</td>
</tr>
<tr>
<td>0.5</td>
<td>2.97</td>
</tr>
<tr>
<td>0.6</td>
<td>1.92</td>
</tr>
<tr>
<td>0.7</td>
<td>1.19</td>
</tr>
<tr>
<td>0.8</td>
<td>0.63</td>
</tr>
<tr>
<td>0.9</td>
<td>0.28</td>
</tr>
<tr>
<td>1.0</td>
<td>0.00</td>
</tr>
</tbody>
</table>

If one uses the geometrical well data \( l/m \) and \( m/r \) then the resistance \( \xi \) can be applied to refine the transmissivity \( T \), as follows:

\[
T = T_{\text{min}}(1 + \xi /10) = cT_{\text{min}} \tag{9}
\]

For LAMO, the typical values of \( l/m \) and \( m/r \) are within the limits: \( 0.5 > l/m > 0.2; \) \( 500 > m/r > 100. \) Then, as it follows from Table II, the correction \( c \) may be within the limits: \( 2.78 > c > 1.29. \)

### III. OBTAINING OF PERMEABILITY MAPS

It was explained before that the variable permeability \( k \)-maps must be used to control the \( T \)-maps of the GV system:

\[
k = T/m \tag{10}
\]

where \( T \) is the transmissivity derived from the well pumping data, \( m \) is the aquifer thickness which is computed and used by the GV system to obtain \( T = km \) of (2).

By using the EXCEL program [6] the set of the specific capacity \( q \) [litre/(sec.meter)] must be extracted from the well pumping data. As a rule the \( q \)-set contains very low and also very high values. In order to normalize the set, minimal and maximal values of \( q \) are fixed. The \( q \)-set contains \( n \) pointwise data. For LAMO \( n > 1000 \) for practically all aquifers. Due to the large \( n \) the very fast gridding method of “inverse distance to power” is applied by the SURFER program [7]. This method computes the interpolated value \( \sigma_o \) at the node by using the available neighboring pointwise data \( \sigma_i = 137.5 \ q_i, \) \( i = 1,\ldots, n, \) as follows [8]:

\[
\sigma_o = \left( \frac{\sum_i \sigma_i r_i}{\sum_i r_i} \right)^p, \quad d_o = \sqrt{(x_o - x_i)^2 + (y_o - y_i)^2} \tag{11}
\]

where \( r_i \) weight of \( \sigma_i, \) \( d_i \) distance between the grid node \( o \) and the \( i \)-th data location point; \( p \) weighting power; \( x_o, y_o; \) \( x_i, y_i \) coordinates, respectively, of the \( o \)-th grid node and the \( i \)-th point. The value \( p = 2 \) is used, to prepare the data for LAMO.

The interpolation result of (11) is rather rough and, to smooth it, the moving digital “inverse distance” low-pass filter of size \( 11 \times 11 \) was used [9]:

\[
\sigma_{o'} = \left( \frac{\sum_{i=0}^{10} \sigma_i r_i}{\sum_{i=0}^{10} r_i} \right)^p, \quad \tau = (1/D_o)^p, \quad D_o = \sqrt{i^2 + j^2} \tag{12}
\]

where \( \tau \) filter weight; \( p \) the power (\( p = 0.5 \) is applied); \( i \) and \( j \) the grid row and column local indices for the neighboring nodes with respect to the central node \( oo \) of the filter; \( D_o \) the distance between the nodes \( oo \) and \( ij \).

In Table III the first quadrant of the \( \tau \) matrix of the filter (12) is shown. The filter contains four symmetrical quadrants, because negative \( i \) and \( j \) indices are also applied.
In Table IV the summary of characteristic parameters for the primary aquifers of LAMO are presented. The data for the $k$ and $T$ values are preliminary, because they have been obtained due to rather rough approximations without accounting for the hydraulic resistances of individual wells. No checking of pumping data correctness has been done.

In Table IV the following acronyms are used: $C$ is the code of aquifer; $L_C$ is the area of aquifer [thous.$\text{km}^2$]; $m_{\text{mean}}$ is the mean thickness [meter]; $k_{\text{mean}}, k_{\text{min}}, k_{\text{max}}$ are mean, minimal, maximal permeability [meter/day]; $T_{\text{mean}}, T_{\text{max}}$ are mean, maximal transmissivity [(meter)$^2$/day]; $(l/m)$ is mean parameter that can be used to compute the resistance $\xi$ that is applied for the correction (9).

**IV. CONCLUSION**

Formerly, the constant values represented the permeability $k$-maps of LAMO. By using the information of well pumping, the upgraded variable $k$-maps have been created. To obtain them, the digital interpolation and image processing methods were applied. Currently the new $k$-maps are used in LAMO that results in refined transmissivity distributions of the HM primary aquifers. To make these results better the well geometrical data will be accounted for and the initial pumping data will be checked and corrected.

**ACKNOWLEDGMENT**

In 2010 – 2012 the hydrogeological model of Latvia LAMO has been developed within the framework of the project “Creating of Hydrogeological Model of Latvia to be Used for Management of Groundwater Resources and for Evaluation of their Recovery Measures”. The project was co-financed by the European Regional Development Fund.

**APPENDIX**

**APPLICATION OF WELL PUMPING DATA FOR CREATING OF THE D3pl AQUIFER $k$-MAP.**

In Fig. 1a the location of wells is shown. They are not distributed evenly within the area of aquifer. Number of wells $n = 1730$; $5.0 > q > 0.3$.

In Fig. 2a the interpolated and smoothed isolines of transmissivity are shown when the “inverse distance” method (11) and the filtering (12) have been applied. The results are existing in all nodes of the full LAMO area grid.

The $m$-map of the aquifer thickness is shown in Fig. 3a. One can notice that the map includes rather deep incisions of the Daugava and Saka river valleys.

In Fig. 4a the initial and filtered $k$-maps are shown. Filtering has eliminated the wrong uplift of the $k$ values at the places of incisions of the Daugava and Saka river valleys.

Formerly the $k$-map for the D3pl aquifer was a constant value. The upgraded $k$-distributions of Fig. 4a are rather variable and, for this reason, the parameter $k_{\text{mean}}$ is used as a part of the product (13) for the GV system.

The final refined $T$-map which is currently used by LAMO is shown in Fig. 5a. Due to filtering of the $k$-map at the location of incisions of the Daugava and Saka river valleys the transmissivity is correct.
Fig. 1a. Location of wells for the D3pl aquifer.

Fig. 2a. Interpolated (red) and smoothed (black) isolines of transmissivity.
Fig. 3a. M-map for the aquifer thickness; incisions of the Daugava and Saka river valleys.

Fig. 4a. Initial and filtered k-maps (initial and filtered lines are red and black); filtering has eliminated the wrong uplift of the k values at the places of incisions of the Daugava and Saka river.
Fig. 5a. Final T-map. Places of the Daugava and Saka river incisions and transmissivity drops.

REFERENCES


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